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INFLUENCE OF MATERIAL INHOMOGENEITIES ON THE STRESS DISTRIBUTION NEAR A THIN ELASTIC INCLUSION*

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A solution is obtained for the plane problem of elasticity theory for a thin-walled elastic inclusion in a matrix whose shear modulus depends exponentially on the coordinate that coincides with the axial line of the interlayer.

A solution of the problem for a crack in a plane with shear modulus $\mu(y) = \mu(1 + c|y|)$, $c = \text{const}$ /1/ and with a shear modulus $\mu(x) = \mu e^{\beta x}$, $\beta = \text{const}$ /2/ has been constructed earlier.

1. Formulation of the problem. The elastic equilibrium of an inhomogeneous plane containing a thin inhomogeneous elastic inclusion of small thickness $2h$ (Fig.1) on a segment $[-a, a]$ of the x axis is considered. The matrix material possesses the variable shear modulus $\mu(x) = \mu e^{\beta x}$, $\beta = \text{const}$ and the constant Poisson's ratio ν, and for the inclusion material $\mu_0(x) = \mu_0 e^{\beta_0 x}$, $\beta_0 = \text{const}$ and ν_0, respectively. The mechanical contact between the inclusion and the matrix inhomogeneity is characterized by total adhesion. The composite body is under the conditions of the plane problem subjected to a uniform tensile load at infinity.

When taking account of the small thickness of the inclusion and the symmetry of the problem, the interaction of a thin elastic inclusion with the surrounding medium can be described by the system of differential relationships /3, 4/

$$\begin{gathered}2\mu_0(x)\, u_{,x}(x, +0) = c_{10} N(x) - c_{20}\sigma_y(x, +0)\\ 2\mu_0(x)\, v(x, +0) = h\,[c_{10}\sigma_y(x, +0) - c_{20} N(x)]\\ N(x) = N(-a) - \frac{1}{h}\int_{-a}^{x} \sigma_{xy}(t, +0)\, dt, \quad -a \leqslant x \leqslant a\\ c_{10} = (1 + \varkappa_0)/4, \quad c_{20} = (3 - \varkappa_0)/4\end{gathered} \tag{1.1}$$

$\varkappa_0 = (3 - \nu_0)/(1 + \nu_0)$ for the generalized plane state of stress $\varkappa_0 = 3 - 4\nu_0$ for the plane strain, and $N(-a)$ is the normal force on the endface $x = -a$ of the inclusion.

2. Method of solution. By virtue of the linearity, the solution of the problem under investigation can be represented in the form

$$(\sigma, u) = (\sigma^\circ, u^\circ) + (\sigma^*, u^*) \tag{2.1}$$

where $\sigma^\circ = (\sigma_x^\circ, \sigma_y^\circ, \sigma_{xy}^\circ)$ is the stress, $u^\circ = (u^\circ, v^\circ)$ is the displacement due to a given external load in an inhomogeneous plane without an inclusion (the fundamental problem) and $\sigma^* = (\sigma_x^*, \sigma_y^*, \sigma_{xy}^*)$, $u^* = (u^*, v^*)$ are the stress and displacement for the perturbed part of the problem. Assuming the quantities σ°, u° to be known, we can determine the values σ^*, u^*. Let $U(x, y)$ be the stress function for the perturbed part of the problem. Then

$$\sigma_x^* = U_{,yy}, \quad \sigma_y^* = U_{,xx}, \quad \sigma_{xy}^* = -U_{,xy} \tag{2.2}$$

Substituting (2.2) into the strain compatibility equation taking Hooke's law into account we obtain the differential equation

$$\begin{gathered}\nabla^4 U - 2\beta\nabla^2 U_{,x} + \beta^2(U_{,xx} - cU_{,yy}) = 0\\ c = c_2/c_1, \quad c_1 = (1 + \varkappa)/4, \quad c_2 = (3 - \varkappa)/4\end{gathered} \tag{2.3}$$

$\varkappa = (3 - \nu)/(1 + \nu)$ for the generalized plane state of stress, $\varkappa = 3 - 4\nu$ for plane strain, and ∇^2 is the Laplace operator. We will seek the function $U(x, y)$ in the form of the Fourier integral

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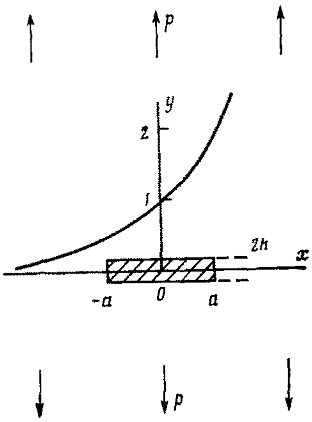

Fig.1

$$U(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{U}(\alpha, y) e^{-ix\alpha} d\alpha, \quad -\infty < x < \infty, \quad y > 0 \tag{2.4}$$

$$\overline{U}(\alpha, y) = \int_{-\infty}^{\infty} U(x, y) e^{ix\alpha} dx, \quad -\infty < \alpha < \infty, \quad y > 0$$

We then obtain from (2.3)

$$\frac{d^4\overline{U}}{dy^4} + (2i\beta\alpha - 2\alpha^2 - c\beta^2)\frac{d^2\overline{U}}{dy^2} + (\alpha^4 - 2i\beta\alpha^3 - \beta^2\alpha^2)\overline{U} = 0 \tag{2.5}$$

The general solution of (2.5) that decreases at infinity has the form

$$U(\alpha, y) = \mathbf{A}_1(\alpha) e^{-m_1 y} + \mathbf{A}_2(\alpha) e^{-m_2 y}, \quad y \geqslant 0 \tag{2.6}$$
$$m_1 = ((-\gamma_1 + \gamma_2)/2)^{1/2}, \quad m_2 = ((-\gamma_1 - \gamma_2)/2)^{1/2}$$
$$\gamma_1 = 2i\beta\alpha - 2\alpha^2 - c\beta^2, \quad \gamma_2 = (\beta^4 c^2 - 4i\beta^3 c\alpha + 4\beta^2 c\alpha^2)^{1/2}$$

($\mathbf{A}_1(\alpha)$, $\mathbf{A}_2(\alpha)$ are complex functions). We find

$$\sigma_x^*(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum \mathbf{A}_j(\alpha) m_j^2 e^{-m_j y} e^{-ix\alpha} d\alpha \tag{2.7}$$

$$\sigma_y^*(x, y) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \alpha^2 \sum \mathbf{A}_j(\alpha) e^{-m_j y} e^{-ix\alpha} d\alpha$$

$$\sigma_{xy}^*(x, y) = -\frac{i}{2\pi} \int_{-\infty}^{\infty} \alpha \sum \mathbf{A}_j(\alpha) m_j e^{-m_j y} e^{-ix\alpha} d\alpha$$

$$2\mu(x) u_{,x}^*(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum l_{2j} \mathbf{A}_j(\alpha) e^{-m_j y} e^{-ix\alpha} d\alpha$$

$$2\mu(x) v_{,x}^*(x, y) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} (\beta + i\alpha) \sum \frac{l_{1j}}{m_j} \mathbf{A}_j(\alpha) e^{-m_j y} e^{-ix\alpha} d\alpha$$
$$l_{1j} = c_1\alpha^2 + c_2 m_j^2, \quad l_{2j} = c_2\alpha^2 + c_1 m_j^2, \quad j = 1, 2$$

from relations (2.2), (2.4), (2.6) and Hooke's law.

Here and everywhere henceforth, unless otherwise stated, the summation is over j from $j = 1$ to $j = 2$.

Let us use the notation

$$\sigma_{xy}^*(x, +0) = f_1(x), \quad 2\mu(x) v_{,x}^*(x, +0) = f_2(x), \quad -\infty < x < \infty \tag{2.8}$$

where $f_j(x) = 0, \ |x| > a$. Taking (2.7) into account, we obtain from relationships (2.8)

$$\mathbf{A}_1(\alpha) = -\frac{m_1 l_{12}}{i\alpha^3 c_1 m} F_1(\alpha) + \frac{m_1 m_2^2}{(\beta + i\alpha)\alpha^2 c_1 m} F_2(\alpha) \tag{2.9}$$

$$\mathbf{A}_2(\alpha) = \frac{m_2 l_{11}}{i\alpha^3 c_1 m} F_1(\alpha) - \frac{m_2 m_1^2}{(\beta + i\alpha)\alpha^2 c_1 m} F_2(\alpha)$$

$$F_j(\alpha) = \int_{-a}^{a} f_j(t) e^{i\alpha t} dt, \quad j = 1, 2; \quad m = m_1^2 - m_2^2$$

Substituting the functions $\mathbf{A}_1(\alpha)$, $\mathbf{A}_2(\alpha)$ into (2.7) and passing to the limit $y \to +0$ we obtain

$$\sigma_y^*(x, +0) = \lim_{y \to +0} \left[\frac{1}{2\pi} \int_{-a}^{a} \sum f_j(t) h_{1j}(x, y, t)\, dt\right] \tag{2.10}$$

$$2\mu(x) u_{,x}^*(x, +0) = \lim_{y \to +0} \left[\frac{1}{2\pi} \int_{-a}^{a} \sum f_j(t) h_{2j}(x, y, t)\, dt\right], \quad -\infty < x < \infty$$

Here

$$h_{kj}(x, y, t) = \int_{-\infty}^{\infty} [\mathbf{H}_{kj}(\alpha, y) - \mathbf{H}_{kj}^{\infty}(\alpha, y)]\, e^{i\alpha(t-x)} d\alpha + \tag{2.11}$$

$$\int_{-\infty}^{\infty} \mathbf{H}_{kj}^{\infty}(\alpha, y)\, e^{i\alpha(t-x)}\, d\alpha; \quad \lim_{|\alpha|\to\infty} \frac{\mathbf{H}_{kj}^{\infty}(\alpha, y)}{\mathbf{H}_{kj}(\alpha, y)} = 1$$

$$\mathbf{H}_{11}(\alpha, y) = \frac{m_1 l_{12} e^{-m_1 y} - m^2 l_{11} e^{-m_2 y}}{i\alpha c_1 m}, \quad \mathbf{H}_{12}(\alpha, y) = \frac{m_1 m_2 (-m_2 e^{-m_1 y} + m_1 e^{-m_2 y})}{(\beta + i\alpha) c_1 m}$$

$$\mathbf{H}_{21}(\alpha, y) = \frac{-m_1 l_{12} l_{21} e^{-m_1 y} + m_2 l_{11} l_{22} e^{-m_2 y}}{i\alpha^3 c_1 m}$$

$$\mathbf{H}_{22}(\alpha, y) = \frac{m_1 m_2 (m_2 l_{21} e^{-m_1 y} - m_1 l_{22} e^{-m_2 y})}{(\beta + i\alpha)\alpha^2 c_1 m}$$

As follows from (2.6), as $|\alpha| \to \infty$ we have $m_j \to |\alpha|$, and relationships (2.11) result in the equalities

$$\mathbf{H}_{kj}^{\infty}(\alpha, y) = \lambda_{kj} \frac{\alpha}{i|\alpha|} e^{-|\alpha| y}, \quad k, j = 1, 2 \tag{2.12}$$

$$\lambda_{11} = \frac{c_1 - c_2}{2c_1}, \quad \lambda_{12} = \frac{1}{2c_1}, \quad \lambda_{21} = \frac{c_2^2 - 2c_1 c_2 - 3c_1^2}{2c_1}, \quad \lambda_{22} = \frac{c_1 - c_2}{2c_1}$$

Taking account of the value of the integral

$$\frac{1}{2i} \int_{-\infty}^{\infty} \frac{\alpha}{|\alpha|} e^{-|\alpha| y} e^{i\alpha(t-x)} d\alpha = \frac{t - x}{(t-x)^2 + y^2}$$

we obtain from relationships (2.1), (2.10), (2.12)

$$\sigma_y(x, +0) = \sigma_y^{\circ}(x) + \frac{1}{\pi} \int_{-a}^{a} \sum f_j(t) \left[\frac{\lambda_{1j}}{t - x} + k_{1j}(x, t)\right] dt \tag{2.13}$$

$$2\mu(x) u_{,x}(x, +0) = 2\mu(x) \overset{\circ}{u}_{,x}(x) + \frac{1}{\pi} \int_{-a}^{a} \sum f_j(t) \left[\frac{\lambda_{2j}}{t - x} + k_{2j}(x, t)\right] dt$$

$$k_{kj}(x, t) = \int_{0}^{\infty} \{\mathrm{Re}\,[\mathbf{M}_{kj}(\alpha) e^{i\alpha(t-x)}] - \lambda_{kj} \sin(t - x)\alpha\}\, d\alpha, \quad k, j = 1, 2$$

$$\mathbf{M}_{11}(\alpha) = \frac{n_1}{i\alpha c_1 m_+}, \quad \mathbf{M}_{12}(\alpha) = \frac{m_1 m_2}{(\beta + i\alpha) c_1 m_+}, \quad \mathbf{M}_{22}(\alpha) = -\frac{n_2}{\alpha^2} \mathbf{M}_{12}(\alpha)$$

$$\mathbf{M}_{21}(\alpha) = -\frac{c_1 c_2 \alpha^4 + c_1^2 \alpha^2 (m_1^2 + m_1 m_2 + m_2^2) - c_2 m_1 m_2 n_2}{i\alpha^3 c_1 m_+}$$

$$m_+ = m_1 + m_2, \quad n_1 = c_1 \alpha^2 - c_2 m_1 m_2, \quad n_2 = c_2 \alpha^2 - c_1 m_1 m_2$$

Substituting (2.13) into the conditions for interaction between a thin-walled elastic inclusion and an inhomogeneous matrix (1.1) and changing to dimensionless quantities, we obtain a system of two singular integral equations

$$\frac{1}{\pi} \int_{-1}^{1} \sum \varphi_j(\tau) \left[\frac{a_{kj}}{\tau - \xi} + b_{kj} \operatorname{sign}(\xi - \tau) + \mathbf{K}_{kj}(\xi, \tau)\right] d\tau = \Phi_k(\xi) \tag{2.14}$$

$$k = 1, 2; \; -1 < \xi < 1$$

Here

$$\mathbf{K}_{1j}(\xi, \tau) = \lambda_2 k_{1j}(a\xi, a\tau) + k_{2j}(a\xi, a\tau)$$

$$\mathbf{K}_{2j}(\xi, \tau) = \lambda_1 k_{1j}(a\xi, a\tau), \quad \varphi_j(\tau) = f_j(a\tau)$$

$$a_{1j} = \lambda_2 \lambda_{1j} + \lambda_{2j}, \quad a_{2j} = \lambda_1 \lambda_{1j}, \quad \lambda_j = c_j^{\circ}/k, \quad j = 1, 2$$

$$\Phi_1(\xi) = -\lambda_2 \sigma_y^{\circ}(a\xi) - 2\mu(a\xi) \overset{\bullet}{u}_{,x}(a\xi) + {}^1/_2 [N(a) + N(-a)]$$

$$\Phi_2(\xi) = -\lambda_1 \sigma_y^\circ(a\xi) + {}^1/_2[N(a) + N(-a)] + {}^1/_2 h_0^{-1}[V(a) + V(-a)]$$
$$b_{11} = \frac{\pi\lambda_1}{2h_0}, \quad b_{12} = 0, \quad b_{21} = \frac{\pi\lambda_2}{2h_0}, \quad b_{22} = -\frac{\pi}{2h_0}$$
$$N(w) \equiv {}^1/_2[\sigma_x(w, h) + \sigma_x(w, -h)], \quad V(w) \equiv \mu(w)[v(w, h) - v(w, -h)]$$
$$k = \frac{\mu_0(a\xi)}{\mu(a\xi)}, \quad h_0 = \frac{h}{a}, \quad \xi = \frac{x}{a}, \quad \tau = \frac{t}{a}, \quad w = \pm a$$

The required functions $\varphi_1(\tau)$, $\varphi_2(\tau)$ satisfy the additional conditons

$$\int_{-1}^{1} \varphi_j(\tau)\,d\tau = \mathbf{B}_j, \quad j = 1, 2 \tag{2.15}$$
$$\mathbf{B}_1 = h_0[N(-a) - N(a)], \quad \mathbf{B}_2 = V(a) - V(-a)$$

A priori formulas for calculating the axial force $N(w)$ and the relative vertical displacement $V(w)$ on the endfaces $w = \pm a$ of the inclusion are presented in /4/.

The system of singular integral Eqs.(2.14) and (2.15) describes the elastic equilibrium of a plane with an inclusion of arbitrary stiffness: from absolutely compliant (crack) to absolutely rigid. In the case when the mechanical parameters of the matrix and inclusion are equal, it follows from the system of integral Eqs.(2.14) and (2.15) that $\varphi_i(\tau) = 0$, $(i = 1, 2)$ and therefore, the absence of perturbations induced by the inclusion. For $k = 0$ we have $\varphi_1(\tau) = 0$ and the integral equation

$$\frac{1}{\pi}\int_{-1}^{1} \varphi_2(\tau)\left[\frac{\lambda_{12}}{\tau - \xi} + k_{12}(\xi, \tau)\right]d\tau = -\sigma_y^\circ(a\xi), \quad -1 < \xi < 1$$

for a crack in an inhomogeneous plate, obtained earlier /2/. If $k \to \infty$, then $\varphi_2(\tau) = 0$ follows from (2.14) and

$$\frac{1}{\pi}\int_{-1}^{1} \varphi_1(\tau)\left[\frac{\lambda_{21}}{\tau - \xi} + k_{21}(\xi, \tau)\right]d\tau = -2\mu(a\xi)\,\overset{\circ}{u}_{,x}(a\xi), \quad -1 < \xi < 1$$

is the integral equation for an absolutely rigid inclusion in an inhomogeneous plane. Setting $\beta = \beta_0 = 0$, we obtain the solution for a thin-walled elastic inclusion in a homogeneous matrix /5/.

3. ***Numerical analysis.*** We will represent the solution of the system of integral Eqs. (2.14) and (2.15) in the form

$$\varphi_j(\tau) = G_j(\tau)/\sqrt{1 - \tau^2}, \quad -1 < \tau < 1, \quad j = 1, 2 \tag{3.1}$$

($G_j(\tau)$ is a bounded measurable function). Substitution of the representation (3.1) into the integral Eqs.(2.14) and (2.15) and utilization of the analogue of the Lobatto-Jacobi quadrature formula for singular integrals /6/ lead to a system of linear algebraic equations

$$\sum_{r=1}^{n} W_r \sum_{j=1}^{2} G_j(\tau_r)\left[\frac{a_{kj}(\xi_l)}{\tau_r - \xi_l} + b_{kj}\operatorname{sign}(\xi_l - \tau_r) + \mathrm{K}_{kj}(\xi_l, \tau_r)\right] =$$
$$\Phi_k(\xi_l); \quad l = 1, 2, \ldots, n - 1; \quad k = 1, 2 \tag{3.2}$$
$$\sum_{r=1}^{n} W_r G_j(\tau_r) = \mathbf{B}_j/\pi, \quad j = 1, 2$$

Here

$$W_1 = W_n = \frac{1}{2(n-1)}, \quad W_r = \frac{1}{1-n}, \quad r = 2, 3, \ldots, n - 1$$
$$\tau_r = \cos\left(\frac{r-1}{n-1}\pi\right), \quad r = 1, 2, \ldots, n; \quad \xi_l = \cos\left(\frac{2l-1}{2n-2}\pi\right), \quad l = 1, 2, \ldots, n - 1$$

The system of linear algebraic Eqs.(3.2) was solved numerically in the case of equality of the matrix and inclusion inhomogeneity parameters $\beta = \beta_0$ and $\mu \neq \mu_0$. The solution of the fundamental problem has the form

$$\sigma_y^\circ = p, \quad 2\mu(x)\, u^\circ_{,x} = -c_1 p$$

and $B_j = 0$ ($j = 1, 2$) in (2.15). The state of stress in the neighbourhood of the vertex of a thin elastic inclusion is characterized by the stress intensity factor

$$k^I(w) = \lim_{x \to w} \sqrt{2(x-w)}\, \sigma_y(x, w), \quad w = \pm a \tag{3.3}$$

Substituting the value $\sigma_y(x, +0)$ from relation (2.13) into (3.3), we obtain

$$k^I(\pm a) = p\sqrt{a}\,[k_1^I(\pm a) + k_2^I(\pm a)], \quad k_j^I(\pm a) = \mp \lambda_{1j} G_j(\pm 1), \quad j = 1, 2 \tag{3.4}$$

The vertical displacement of the edges of an inclusion was additionally investigated for compliant inclusions ($k < 1$), while the distribution of shear stresses on the inclusion axis was studied for stiffer inclusions as compared with the matrix ($k > 1$). The following interpolation formula was used here /6/

$$G_j(\tau) = \frac{1+\tau}{2} G_j(1) + \frac{1-\tau}{2} G_j(-1) + (1-\tau^2) \sum_{m=0}^{n-3} b_m^j U_m(\tau)$$
$$b_m^j = \frac{2}{n-1} \sum_{r=2}^{n-1} \left[G_j(\tau_r) - \frac{1+\tau_r}{2} G_j(1) - \frac{1-\tau_r}{2} G_j(-1) \right], \quad j = 1, 2 \tag{3.5}$$

($U_m(\cdot)$ are Chebyshev polynomials of the second kind). Then

$$\tau_{xy}(x, +0) = (a^2 - x^2)^{-1/2} G_1(x/a)$$

and integrating the second of relationships (2.8) with respect to x while taking account of (3.1), we have

$$2\mu(x) \frac{v(x)}{a} = \int_{-1}^{\xi} \frac{G_2(\tau)\, d\tau}{\sqrt{1-\tau^2}} + \frac{V(-a)}{a} \tag{3.6}$$

Substituting the value of $G_2(\tau)$ from (3.5) into relation (3.6) and evaluating the integral, we find

$$2\mu(x) \frac{v(x)}{a} = \frac{1}{2}\left(\arcsin\xi + \frac{\pi}{2} - \sqrt{1-\xi^2}\right) G_2(1) + \tag{3.7}$$

$$\frac{1}{2}\left(\arcsin\xi + \frac{\pi}{2} + \sqrt{1-\xi^2}\right) G_2(-1) + \frac{b_0^2}{2}\left(\arcsin\xi + \frac{\pi}{2} + \xi\sqrt{1-\xi^2}\right) +$$
$$\sqrt{1-\xi^2} \sum_{m=1}^{n-3} b_m^2 \left(\frac{U_{m+1}(\xi)}{m+2} - \frac{U_{m-1}(\xi)}{m} \right) + \frac{V(-a)}{a}$$

Computations were performed for the case of a generalized plane state of stress for $h_0 = 0.1$, $\nu = \nu_0 = 0.3$. It is required to take $n = 5$ for the most unfavourable parameters of the problem to achieve a 1% relative accuracy.

Table 1

k	K_1^+	K_2^+	K_1^-	K_2^-
10^{-4}	−0	1093	−0	898
10^{-2}	−4	896	−3	772
10^{-1}	−14	345	−11	320
10	73	1	64	1
10^2	112	0	95	0
10^4	118	0	100	0

The dependence of the stres intensity factors on the inhomogeneity parameter β turns out to be almost linear for $k = 10^{-3}$ ($j = 2$) and $k = 10^2$ ($j = 1$)

$$K_1^\pm = 111 + 10\beta, \quad K_2^\pm = 980 + 100\beta$$
$$(K_j^\pm = k_j^I(\pm a)\, p^{-1} a^{-1/2} \cdot 10^3, \quad j = 1, 2)$$

Table 2

β	k = 0,1				k = 10	
	K_1^+	K_2^+	K_1^-	K_2^-	K_1^+	K_1^-
0	—12	335	—12	335	69	69
0.2	—13	340	—11	329	71	67
0.4	—13	343	—11	323	72	65
0.6	—14	346	—10	317	73	63
0.8	—15	348	—10	310	74	61
1	—16	350	—10	304	75	59

The dependence of these quantities on the relative stiffness k of the inclusion is given in Table 1 for $\beta = 0.5$. Values of $K_j^{\pm}$ $(j = 1, 2)$ as a function of the inhomogeneity parameter β are presented in Table 2 for $k = 0.1$ and $k = 10$ (in the latter case $K_2^{\pm} = 1$ for all β).

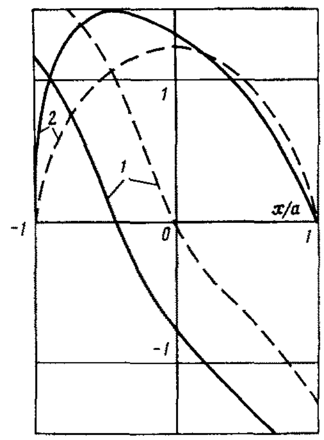

Fig.2

The distribution of the shear stresses $(a^2 - x^2)^{1/2}\sigma_{xy}(x/a)\cdot 10$ over the axial line $-a < x < a$, $y = 0$ of the inclusion is shown in Fig.2 for $k = 10^3$ (curves *1*) and the normal displacements $2\mu(x)v(x)/(ap)$ found by means of (3.7) for a compliant inclusion for $k = 10^{-3}$ (curves *2*) for $\beta = 0$ (the solid line) and $\beta = 0.5$ (the dashed line).

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